

Novel Noise Reduction Using DTCWT Based Shrinkage with RSR and RACE

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Abstract: in this research work, it has been proposed to denoise natural images employing Dual Tree Wavelet Transform (DTCWT). In this paper, emphasis is on removing noise in natural images of various sized dimensions. The dual-tree complex wavelet transform is a relatively recent enhancement to the discrete wavelet transform (DWT), which allows for distinction of data directionality in the transform space. In extension, Random Spray Retinex (RSR) is introduced by image enhancement methods. FNLM is a recent denoising method which has been added for denoising an image. Across the six orientations of the DTWCT the standard deviation of non-enhanced image coefficients can be computed, and then it normalized for each level of the transform. The result is a map of the directional structures present in the non-enhanced image. Then said sampling map is used to shrink the coefficients of the enhanced image. According to data directionality the shrunk coefficients and the coefficients of the non-enhanced image are mixed. Finally the obtained denoising results using inverse DTWCT and FNLM are compared with other transforms such as Steerable Pyramid Transform (SPT), Coutourlet Transform, Shearlet Transform, and DWT for noise reduction. The Performance parameters PSNR and SSIM have been taken into account and the noise is reduced significantly superior from original image signal, compared to conventional denoising methods.

Index Terms—Dual-tree complex wavelet transforms (DTWCT), noise reduction, image enhancement, random sprays, and shrinkage.

INTRODUCTION

The dual-tree complex wavelet transform (CWT) is a relatively recent enhancement to the discrete wavelet transform (DWT) [1], with important additional properties: It is nearly shift invariant and directionally selective in two and higher dimensions. Although the field of image enhancement has been active since before digital imagery achieved a consumer status, it has never stopped evolving. If we use image enhancement algorithms based on random spray sampling a specific image quality problems are raised to remove that this paper introduces a novel multi-resolution denoising method. While inspired by the peculiar problem of such Method, the proposed approach for other image enhancement methods that either introduce or

exacerbate noise. This work builds and expands based on a previous article by Fierro et al. [2]. Random sprays are a two-dimensional collection of points with a given spatial distribution around the origin. Sprays can be used to sample an image support in place of other techniques, and have been previously used in works such as Provenzi et al [3].

Random sprays have been partly inspired by the Human Visual System (HVS). In particular, a random spray is not dissimilar from the distribution of photo receptors in the retina, although the underlying mechanisms are vastly different. However, due to the peaked nature of sprays, undesired noise is often introduced in the output images generated by image enhancement algorithms that use sprays as image scanning structures. The statistical characteristics of such noise depend on several factors, including image content and spray properties.

Multi-resolution, transform space denoising methods has a long history. A particular branch is that of transform space coefficients shrinkage, i.e. the magnitude reduction of the transform coefficients according to certain criteria. Such an approach is often referred to as shrinkage. The most commonly used transforms for shrinkage-based noise reduction are the Wavelet Transform (WT) [4], the Steerable Pyramid Transform [5], the Contourlet Transform [6] and the Shear let transform [7]. With the exception of the WT, all other transforms lead to over-complete data representations. It is usually associated with the ability to distinguish data directionality in the transform space.

Independent of the specific wavelet transform used in multi-resolution method. The DTWCT shrinkage coefficient is that image data gives rise to sparse coefficients in the transform space. Such a process can usually be improved by an elaborate statistical analysis of the dependencies between coefficients at different scales. Traditional multi-resolution methods are designed to only remove one particular type of noise (e.g. Gaussian noise). Furthermore, the input image is assumed to the unknown statistical properties of the noise introduced by the use of sprays. The traditional approaches do not find the expected conditions, and their action becomes much less effective.

The proposed approach still performs noise reduction via transform space coefficient shrinkage, yet an element of novelty is introduced, in the form of partial reference images.

Having a reference allows the shrinkage process to be data-driven. A strong source of inspiration were the works on the Dual-tree Complex Wavelet Transform by Kingsbury [8], the work on the Steerable Pyramid Transform by Simoncelli et al. [9], and the work on Wavelet Coefficient Shrinkage and Johnstone [7]. Fig. 1 depicts the differences between traditional noise-reduction methods and the one proposed.

The remainder of this paper is organized as follows. The Dual-tree Complex Wavelet Transform is introduced in Section II,

Pyramid, by means of the Steerable Pyramid Transform (SPT) [9]. While, the SPT is an over complete representation of data, it grants the ability to appropriately distinguish data orientations as well as being shift-invariant. The SPT is not devoid of problems: filter design can be messy, perfect reconstruction is not possible and computational efficiency can be a concern.

Thus, a further development of the SPT, involving the use of pair of filters to compute the energy response, has been accomplished with the Complex Wavelet Transform (CWT) [12]. Similarly to the SPT, in order to retain the whole Fourier spectrum, the transform needs to be over complete by a factor of 4, i.e. there are 3 complex coefficients for each real one. While the CWT is also efficient, since it can be computed through separable filters, it still lacks of assumption the Perfect Reconstruction property.

Therefore, Kingsbury also reduce noise introduced the Dual-tree Complex Wavelet Transform (DTCWT), which has the added statistical characteristic of Perfect Reconstruction at the cost of approximate shift-invariance [8]. Since the topic is extremely vast, only a brief introduction of the DTCWT is given. The reader is referred to the work by Selesnick et al. [11] for a comprehensive coverage on the DTCWT and the relationship it shares with other transforms. As in the case of filter design for real wavelet transforms, there are various approaches to the design of filters for the dual-tree CWT.

In the following, we describe methods to construct filters satisfying the following desired properties:

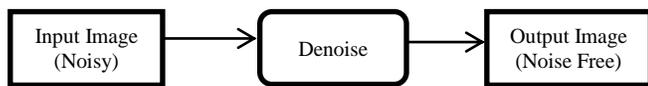
1. approximate half-sample delay property
2. PR (orthogonal or biorthogonal)
3. finite support (FIR filters)
4. vanishing moments/good stop band
5. linear-phase filters

The 2D Dual Tree Complex Wavelet Transform can be implemented using two distinct sets of separable 2D wavelet bases, as shown below.

$$\begin{aligned} \psi_{1,1}(x,y) &= \phi_h(x)\psi_h(y), \psi_{2,1}(x,y) = \phi_g(x)\psi_g(y), \\ \psi_{1,2}(x,y) &= \psi_h(x)\phi_h(y), \psi_{2,2}(x,y) = \psi_g(x)\phi_g(y), \quad (1) \\ \psi_{1,3}(x,y) &= \psi_h(x)\psi_h(y), \psi_{2,3}(x,y) = \psi_g(x)\psi_g(y), \end{aligned}$$

$$\begin{aligned} \psi_{3,1}(x,y) &= \phi_h(x)\psi_h(y), \psi_{4,1}(x,y) = \phi_g(x)\psi_g(y), \\ \psi_{3,2}(x,y) &= \psi_h(x)\phi_h(y), \psi_{4,2}(x,y) = \psi_g(x)\phi_g(y), \quad (2) \\ \psi_{3,3}(x,y) &= \psi_h(x)\psi_h(y), \psi_{4,3}(x,y) = \psi_g(x)\psi_g(y), \end{aligned}$$

Traditional Denoising



Proposed Method

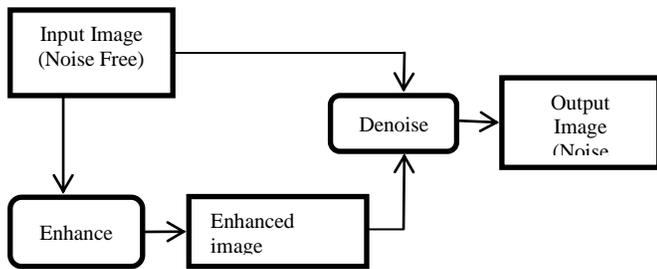


Fig.1. High-level flow charts for traditional noise reduction methods and the proposed one

The concept of Random Spray sampling, and Section III the image enhancement methods Random Spray Retinex and RACE. Section IV then presents the proposed denoising algorithm. The experimental results are presented in Section V, and some final conclusions are drawn in Section VI.

DUAL-TREE COMPLEX WAVELET TRANSFORM

The Discrete Wavelet Transform (DWT) has been a founding stone for all applications of digital image processing: from image denoising to pattern recognition, passing through image encoding and more. While being a complete and (quasi)invertible transform of 2D data, the Discrete Wavelet Transform gives rise to a phenomenon known as “checker board” pattern, which means that data orientation analysis is impossible. Furthermore, the DWT is not shift-invariant, making it less useful for methods based on the computation of invariant features.

To overcome the problems affected by the DWT concept of Steerable filters was introduced by Freeman and Adel son [11], this Steerable filters can used to denoise image into a Steerable

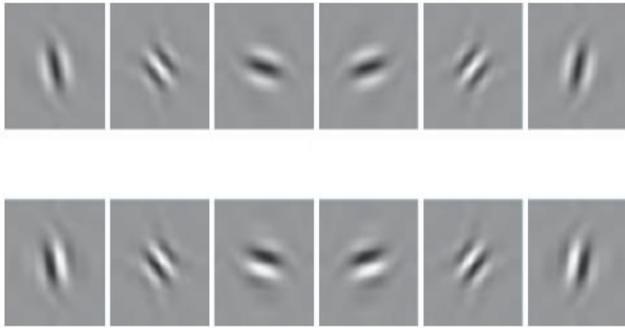


Fig.2. Quasi-Hilbert pairs wavelets used in the Dual Tree Complex Wavelet Transform

When combined, the bases give rise to two sets of real, two-dimensional, oriented wavelets. Fig.2. Quasi-Hilbert pairs wavelets used in the dual-tree complex wavelet transform. Each pair is shown in a column, with the even part on top and the odd one on bottom.

The most interesting characteristic of such wavelets is that they are approximately Hilbert pairs. One can thus interpret the coefficients deriving from one tree as imaginary, and obtain the desired 2D DTCWT. Random Sprays and two image enhancement algorithms that utilize them as sampling structures are introduced in the next Section.

RSR AND RACE

The process of random spray sampling, then introduces Random Spray Retinex (RSR) and RACE, two algorithms that utilize said sampling method. RACE (crasis of RSR and ACE) is the fusion of RSR and an adapted version of Automatic Color Equalization (ACE) [13].

Random Spray Sampling

Define Random Spray sampling was first introduced by Provenzi et al. [14]. Random sprays are an elaboration over the physical spatial scanning structure used by Land in his seminal work on Retinex [15]. In his experiments, Land used a structure resembling a set of paths departing from a central point, on which he mounted a number of photo-detectors. Land's model gave rise to the path-wise family of Retinex algorithms [21], which directly transposed Land's machinery into piece-wise linear paths used to scan the input image. A subsequent thorough mathematical analysis of Retinex [16] allowed the model to be significantly simplified, leading, in turn, to Random Sprays and RSR.

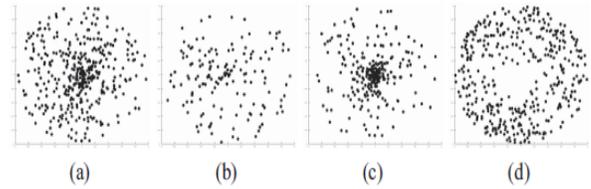


Fig.3. Random sprays with different properties

A single point of a random spray may be generated using the following formulation, and the whole spray is obtained by reiterating the process

$$p = [\rho \cos(\theta), \rho \sin(\theta)] \quad (3)$$

Where $\rho = \text{rand}(0; R)$ and $\theta = \text{rand}(0; 2\pi)$ and rand indicates the uniform random distribution. In particular, R is going to be set as the diagonal of the image, so that the spray can cover its entirety.

The problem of noise

According to the input image, the sharp sampling imposed by sprays leads to the introduction of speckle-like noise with an unknown distribution. The problem of noise reduction has already been partially addressed in the work introducing RACE by using a form of attachment to the original data, thereby strongly noise reducing the appearance of speckles in uniform areas. However, the main problem with the above solution is that to reduce the appearance of noise the enhancement effect is also reduced.

PROPOSED METHOD

The main idea behind this work can be summarized as follows: highly directional content is what conveys the largest part of information to the Human Visual System. This statement is backed up by research, such as the Retinex theory as well as the high-order gray-world assumption (alias gray-edges) [17].

In particular, the local white patch effect described by Retinex comes into play when, for a given channel, the scanning structure samples a positive intensity change in Retinex. For obvious geometrical reasons, intensity changes of a directional nature are more easily crossed than point-like structures such as noise. The proposed method involves around the shrinkage of the real wavelet coefficients generated by the Dual Tree Complex Wavelet Transform, according to data directionality.

Furthermore, since the HVS is highly sensitive to changes in luminance [18], the proposed method first converts the image in a space where the Chroma is separated from the luma (such as YCbCr), and it operates on the wavelet space of the luma channel of both the non-enhanced and enhanced image. While this may seem counter intuitive, since spray based image enhancement algorithms usually operate per channel, the results show vast improvements without visible Contourlet facts. Finally, a fundamental assumption is made: the input image is considered to be either free of noise, or contaminated by non-perceivable noise. If such an assumption holds, the input image contains the information needed for successful noise reduction.

To avoid inappropriate assumptions on the statistical characteristics of noise, a different one is made. In fact, the non-enhanced image is considered to be either free of noise or affected by non-perceivable levels of noise. Taking advantage of the higher sensitivity of the human visual system to changes in brightness, the analysis can be limited to the luma channel of both the non-enhanced and enhanced image.

Also, given the importance of directional content in human vision, the analysis is performed through the dual-tree complex wavelet transform (DTWCT). Unlike the discrete wavelet transform, the DTWCT allows for distinction of data directionality in the transform space. For each level of the transform, the standard deviation of the non-enhanced image coefficients is computed across the six orientations of the DTWCT, and then it is normalized.

The result is a map of the directional structures present in the non-enhanced image. Said map is then used to shrink the coefficients of the enhanced image. The shrunk coefficients and the coefficients from the non-enhanced image are then mixed according to data directionality. Finally, a noise-reduced version of the enhanced image is computed via the inverse transforms. A thorough numerical analysis of the results has been performed in order to confirm the validity of the proposed approach. Random sprays are a two-dimensional collection of points with a given spatial distribution around the origin. Sprays can be used to sample an image support in place of other Techniques.

A. Wavelet coefficients shrinkage

While coefficients associated with non-directional data will have similar energy in all directions. On the other hand, highly directional data will give rise to high energy in one or two directions (although this is not entirely true, as more than two directions may have high energy for “L” or “T” shaped features, but it does not compromise the efficacy of the proposed method).

The luma channels of both the non-enhanced and the enhanced images are transformed using the DTCWT, and the obtained coefficients are elaborated. The output coefficients are transformed into the output image’s luma channel via the inverse DTCWT.

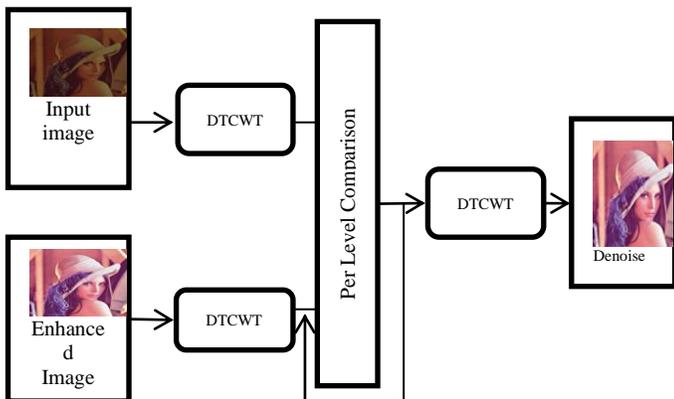


Fig. 4. Proposed method flowchart.

B. Parameter tuning

When dealing with functions with free parameters, a fundamental problem is finding the optimum parameter values. While this can often be attempted with optimization techniques, such methods are unfeasible in the case. To at least provide a reasonable default value for γ_j , the parameter of the Michaelis-Menten function and the depth of the complex wavelet decomposition J , three images from the USC-SIPI Image Database were chosen. Such images provide a good mixture of mostly high-frequency detail (Mandrill), balanced high- and low-frequency content (Lenna), and mainly low frequency content (Splash). The chosen test images are shown in Fig. 5. The main point of novelty is represented by its application in post-processing on the output of an image enhancement method (both the non-enhanced image and the enhanced one are required) and the lack of assumptions on the statistical distribution of noise. On the other hand, the non-enhanced image is supposed to be noise-free or affected by no perceivable noise.

Algorithm 1 for proposed Noise-Reduction Method

```


$$E_{RGB} \leftarrow \text{enhanced}(I_{RGB})$$


$$I_{YCbCr} \leftarrow \text{rgb2ycbcr}(I_{RGB})$$


$$E_{YCbCr} \leftarrow \text{rgb2ycbcr}(E_{RGB})$$


$$Y_1 \leftarrow \text{Y channel of } I_{YCbCr}$$


$$(b^1, c^1) \leftarrow \text{dctwt}(Y_1)$$


$$Y_E \leftarrow \text{Y channel of } E_{YCbCr}$$

    repeat
        
$$(b^E, c^E) \leftarrow \text{dctwt}(Y_E) \quad \Rightarrow \quad (Y_E \text{ is iteration dependent})$$

        for  $j=1 \rightarrow J$  do
            for  $k=1 \rightarrow 6$  do
                
$$e_{j,k} \leftarrow (b_{j,k}^l)^2 + (c_{j,k}^l)^2$$

            end for
            
$$w_j \leftarrow \text{mm}(\text{stddev}(e_{j,k}), \text{median}(e_{j,k}), \gamma_j)$$

            for  $k=1 \rightarrow 6^k$  do
                
$$b_{j,k}^E \leftarrow w_j \cdot b_{j,k}^E + (1-w_j) \cdot b_{j,k}^l$$

                
$$c_{j,k}^E \leftarrow w_j \cdot c_{j,k}^E + (1-w_j) \cdot c_{j,k}^l$$

                
$$i_{j,k}^l \leftarrow \text{ord}(b_{j,k}^l) \quad \Rightarrow \quad (\text{Rank of } b_{j,k}^l)$$

                If  $i_{j,k}^l \in (1, 2)$  then
                    
$$b_{j,k}^l \leftarrow b_{j,k}^E \quad \Rightarrow \quad \text{Shrunk}$$

                coefficients from  $Y_E$ 
                
$$c_{j,k}^O \leftarrow c_{j,k}^E$$

                else
                    
$$b_{j,k}^O \leftarrow b_{j,k}^E \quad \Rightarrow \quad \text{Coefficients from } YI$$

                
$$c_{j,k}^O \leftarrow c_{j,k}^l$$

            end if
        end for
    end for
    end for
    
$$Y_E \leftarrow \text{idctwt}(b^O, c^O) \quad \Rightarrow \quad \text{Inverse DTCWT}$$

    
```

```

until psnr(YI, YE)
until ssim(YI, YE) < 0.001
OYCbCr = concat(YE, ECbCr)
iteration
ORGB = ycbcr2rgb(OYCbCr)
    
```

⇒ YE as in last iteration

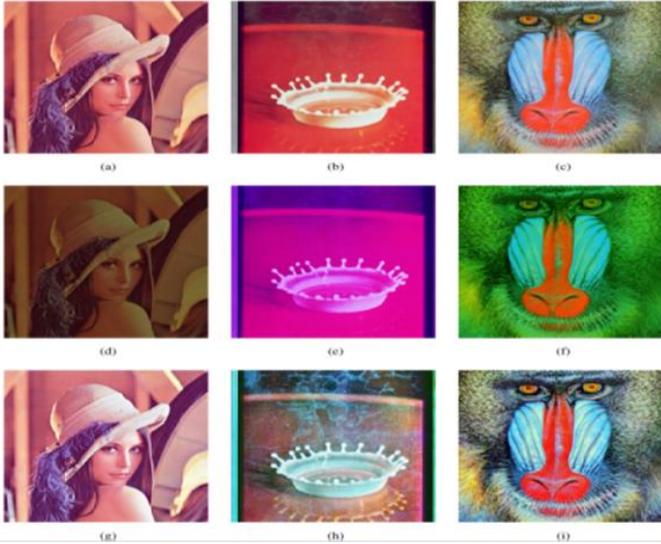


Fig.5. Images from the SIPI database modified to have a strong color cast, then enhanced and used to test the proposed noise reduction method.

Table I: PSNR and SSIM scores for test image from the USC-SIPI image database.

Noise type	Image Name	Noisy		Denoised	
		PSNR	SSIM	PSNR	SSIM
Gaussian	Lenna	27.43	0.58	35.78	0.95
	Splash	27.68	0.49	36.37	0.93
	Mandrill	27.45	0.73	34.78	0.98
Poissonian	Lenna	30.33	0.84	35.78	0.95
	Splash	30.92	0.82	36.37	0.93
	Mandrill	30.16	0.91	34.78	0.98
Speckle	Lenna	27.07	0.55	33.96	0.94
	Splash	27.55	0.53	33.84	0.94
	Mandrill	26.96	0.69	33.87	0.97

Table II: Results of Panel Tests.

The Last Three Columns Represent the Breakdown of Votes for Denoised Images for Different values of J

Question	Image #	Noisy	Denoised	J		
				1	2	3
Least noisy	1	0	12	0	8	3
	2	0	12	3	1	8
	3	0	12	0	0	12
	4	0	2	3	4	5
	5	0	12	8	1	3
Sharpest	1	9	3	3	0	0
	2	6	6	0	4	4
	3	6	6	0	2	2
	4	9	3	0	3	0
	5	12	0	0	0	0
Preferred	1	3	9	2	7	0
	2	3	9	0	3	6
	3	3	9	0	3	6
	4	3	9	3	3	3
	5	7	5	2	0	3

In different rounds, Gaussian, Poissonian and Speckle noise was added to the luma channel of said images and the proposed noise reduction method was run with 3 wavelet levels. The performance was tested using the SSIM [20] measure, holding the unaltered luma channel as the absolute reference. Plots of the results are shown in Fig. 3. The iterations were stopped using a threshold of $t = 0:001$. The SSIM scores and PSNRs are both given in Table I.

Users were asked to select the least noisy, sharpest and preferred image within a set containing the enhanced version of a test picture (using either spray-ACE, RSR or RACE) and the noise reduced images produced with $J = 1, 2$, and 3 . Results are shown in Table II. It can be observed that for all test images users indicated the least noisy among the ones produced by our algorithm. Sharpness, in accordance with the known HVS behavior and the proposed approach characteristics, tends to favor noisy images. Finally, user preference falls mainly on one of the denoised images, indicating good performance.

EXPERIMENTS

To test the proposed method, experiments were performed with three of the images originally used in the works on RSR [20] and RACE which happened to trigger the undesired behavior. The images, their noisy versions (RSR) and the noise reduced results by the proposed approach are all shown in Fig. 4. The images shown in Figs. 4a - 4c were also modified to reduce the dynamic range and introduce a strong color cast. The resulting images were used as test subjects, as illustrated in Fig. 4.

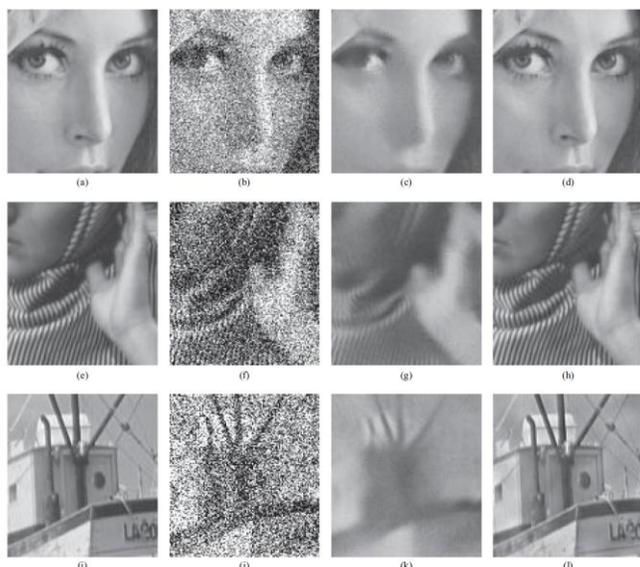


Fig. 6. Comparison between the proposed method and Foveated NL-means.

Retinex-like algorithms have the property of suppressing low frequency gradients much like the HVS, and taking content from the non-enhanced image might negatively affect this behavior. In Fig. 6 we demonstrate that, thanks to the much localized action of the proposed method, one should not worry of reintroducing unwanted gradients.

The proposed method was tested against the Foveated NL-means (FNLM) and BM3D. FNLM by Foi and Boracchi [22] is a recent development of a quite successful work by Buades *et al.* [23]. BM3D [24] is a work by Dabov *et al.* and it is also part of the non-local family of noise reduction methods, although it takes a two-step approach, first computing a rough estimate and then using it to drive Wiener estimation. Finally, the proposed approach was compared to a recent development on the Non Local means approach, namely the FoveatedNL-means. Three images were chosen from the said work, and the proposed noise reduction method used after the addition of Gaussian noise with varying standard deviation. The PSNRs and SSIM scores were both computed and are reported in Tables III and IV: the numbers differ slightly from the original article since the images were generated anew. While the comparison is not entirely fair, as the Foveated NL-means is a reference-less denoising method, it clearly shows the advantage of assuming a partial reference. In order to verify the efficacy of the proposed method, we devised a set of five different tests.

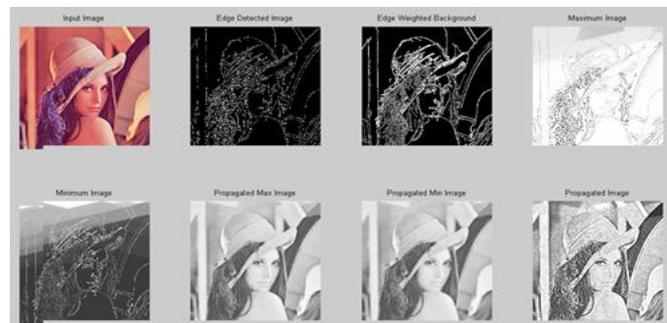


Fig. 7. Image enhancement methods.

To demonstrate the efficacy of the proposed method even in the absence of random spray sampling, a set of images taken from the SIPI database (Girl, Tiffany, and Tree) was enhanced. This also brought out latent grain and compression noises, which were then, reduced using the proposed approach. Fig. 7. To reduce noise introduced or exacerbated by image enhancement methods, in particular algorithms based on the random spray sampling technique.

Performance assessment includes scan line analysis, quality metrics comparison, and subjective evaluation and panel tests.

- Test on Difficult Images for RSR
- Test on Images Enhanced With Histogram Equalization
- Test on Images With Added Color-Cast Enhanced With Spray-Based Methods
- Test Versus Traditional Noise Reduction Method
- User Preference Test

A. Scan line analysis

For a better analysis of the effects of the proposed denoising method, a scan line from two of the test images was selected with the objective of encompassing areas rich in detail, as well as “flat” ones. The scan line data is plotted in Fig. 6a and 6b and on close inspection it becomes apparent how the proposed denoising method acted according to the desired behavior.

TABLE III: Comparison of PSNR for three images taken from the Work on Foveated NL-means.

Σ	Barbara		Boats		Lenna	
	FNLM	Our	FNLM	Our	FNLM	Our
10	33.39	41.71	32.73	42.76	32.39	39.43
20	30.46	37.86	29.89	38.94	35.05	43.24
30	28.08	35.83	28.05	36.93	30.51	37.42
40	26.27	34.52	26.59	35.46	28.91	36.19
50	24.77	33.61	25.24	34.37	27.63	35.11

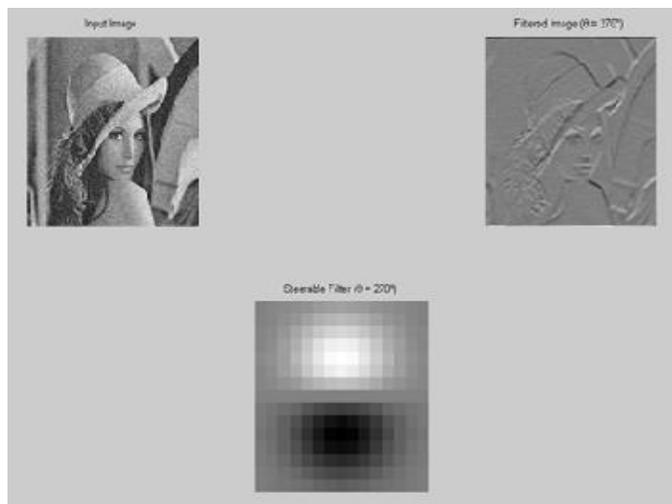


Fig.8. Filter and Algorithm Apply

In order to perform tests against existing methods in literature, it is thus necessary to employ a simplified testing model. The proposed method was tested against the Foveated NL-means (FNLM) and BM3D. It is also part of the non-local family of noise reduction methods, although it takes a two-step approach, first computing a rough estimate and then using it to drive Wiener estimation. The algorithm for the proposed method is given as Algorithm 1. For ease of reference, a visual description is also given in Fig. 8. The following subsections explain the details of the shrinkage process and the tests performed to optimize the algorithm parameters.

In particular Fig. 6g shows how the cover of the blue book has been smoothed out, almost completely removing the noise, yet, at the same time, the letters in the title of the “red dragon” book are still sharp. The same is true for the rest of the text present in the image. This result is also indicative of how the proposed method relies on information present in the non-enhanced image, as the noise reduced output can deviate significantly from the enhanced input, whereas conventional methods will always be limited by the enhanced data.

TABLE IV: Comparison of SSIM scores for three images taken from the Work on Foveated NL-means

Σ	Barbara		Boats		Lenna	
	FNLM	Our	FNLM	Our	FNLM	Our
10	0.97	0.99	0.96	0.99	0.96	0.99
20	0.94	0.98	0.90	0.98	0.93	0.98
30	0.89	0.97	0.84	0.97	0.90	0.96
40	0.85	0.96	0.79	0.95	0.86	0.95
50	0.80	0.94	0.74	0.94	0.83	0.93

CONCLUSIONS

In this paper, we are using wavelet based transforms for image denoising. Noise is one of the major problems in image processing that occurs while capturing the image and the image is transmitting through a channel. Here we are focused on different types of noises like Gaussian, poisson and speckle noises. In this proposed method we have employed DTCWT. The DTCWT has several advantages over traditional denoising methods mainly its shift invariance property and good directional selectivity. The above experimental results have considerably proven that these different images are affected by different type of noises. By using our proposed algorithm we have denoised the noise affected images effectively and also calculated the image quality parameter which is known as PSNR and SSIM value. The proposed method produces good quality output and removing noise without changing the directional structures in the image. Furthermore, we have improved the speed of the algorithm by avoiding iterations and also DTCWT is a tool for other activities such as image quality measures. The Performance parameters PSNR and SSIM have been taken into account and the noise is reduced significantly superior from original image signal, compared to conventional denoising methods.

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