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NANO IDEAL GENERALIZED CLOSED SETS IN NANO IDEAL TOPOLOGICAL SPACES

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Abstract. The purpose of this paper is to introduce a new type of generalized closed and open sets called nIg-closed set and nIg-open set in nano ideal topological spaces and investigate the relation between this set with other sets in nano topological spaces and nano ideal topological spaces. Characterizations and properties of nIg-closed sets and nIg-open sets are given.

1. Introduction

An ideal [7] I on a nonempty set X is a nonempty collection of subsets of X which satisfies (i) $A \in I$ and $B \subset A$ implies $B \in I$ and (ii) $A \in I$ and $B \in I$ implies $A \cup B \in I$. Given a topological space (X, τ) with an ideal I on X and if $P(X)$ is the set of all subsets of X, a set operator $(.)^*\colon P(X) \to P(X)$, called a local function [6] of A with respect to τ and I is defined as follows: for $A \subset X$, $A^*(I, \tau) = \{x \in X\colon U \cap A \notin I$ for every $U \in \tau(x)\}$ where $\tau(x) = \{U \in \tau\colon x \in$ $\in U\}$. A Kuratowski closure operator $cl^*(.)$ for a topology $\tau^*(I, \tau)$, called the $*$-topology, finer than τ is defined by $cl^*(A) = A \cup A^*(I, \tau)$ [13]. When there is no chance for confusion, we will simply write A^* for $A^*(I, \tau)$ and τ^* for $\tau^*(I, \tau)$. If I is an ideal on X, then the space (X, τ, I) is called an ideal topological space. A subset A of an ideal topological space is said to be $*$-dense in itself [4] (resp. $*$-closed [6]) if $A \subset A^*$ (resp. $A^* \subset A$). By a space (X, τ), we always mean a topological space (X, τ) with no separation properties assumed. If $A \subset X$, then $cl(A)$ and $int(A)$ denote the closure and interior of A in (X, τ) respectively, and $int^*(A)$ will denote the interior of A in (X, τ^*). The notion of I-open sets was introduced by Jankovic et al. [5] and it was investigated by Abd El-Monsef [1].

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The notion of nano topology was introduced by Lellis Thivagar [9] which was defined in terms of approximations and boundary region of a subset of an universe using an equivalence relation on it and also defined nano closed sets, nano-interior and nano-closure. K.Bhuvaneswari et al. [2] introduced and studied the concept of nano generalised closed sets in nano topological spaces.

M. Parimala et al. [10, 11, 12] introduced the concept of nano ideal topological spaces and investvestigated some of its basic properties. They also introduced the notion of nI-open sets, nI-closed sets, qnI-open sets and qnI-closed sets in nano ideal topological spaces. In this paper, we introduce a new type of generalized closed and open sets called nIg-closed set and nIg-open set in nano ideal topological spaces and investigate the relationships between this set with other sets in nano topological spaces and nano ideal topological spaces. Characterizations and properties of nIg-closed sets and nIg-open sets are studied.

2. Preliminaries

We recall the following definitions, which will be used in the sequel.

Definition 2.1 ([8]). Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as indiscernibility relation. Then U is divided into disjoint equivalence classes. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space.

Let $X \subseteq U$. Then,

(i) The lower approximation of X with respect to R is the set of all objects which can be for certain classified as X with respect to R and is denoted by $L_R(X)$. That is, $L_R(X) = \cup\{R(X) \colon R(X) \subseteq X, x \in U\}$ where $R(X)$ denotes the equivalence class determined by $x \in U$.

(ii) The upper approximation of X with respect to R is the set of all objects which can be possibly classified as X with respect to R and is denoted by$U_R(X)$. That is, $U_R(X) = \cup\{R(X) \colon R(X) \cap X \neq \phi, x \in U\}$.

(iii) The boundary region of X with respect to R is the set of all objects which can be classified neither as X nor as not -X with respect to R and is denoted by $B_R(X)$. $B_R(X) = U_R(X) - L_R(X)$.

Property 2.2 ([9]). If (U, R) is an approximation space and $X, Y \subseteq U$, then

(i) $L_R(X) \subseteq X \subseteq U_R(X)$
(ii) $L_R(\phi) = U_R(\phi) = \phi$
(iii) $L_R(U) = U_R(U) = U$
(iv) $U_R(X \cup Y) = U_R(X) \cup U_R(Y)$
(v) $U_R(X \cap Y) \subseteq U_R(X) \cap U_R(Y)$
(vi) $L_R(X \cup Y) \supseteq L_R(X) \cup L_R(Y)$
(vii) $L_R(X \cap Y) = L_R(X) \cap L_R(Y)$
(viii) $L_R(X) \subseteq L_R(Y)$ and $U_R(X) \subseteq U_R(Y)$ whenever $X \subseteq Y$
(ix) $U_R(X^c) = [L_R(X)]^c$ and $L_R(X^c) = [U_R(X)]^c$
(x) $U_R[U_R(X)] = L_R[U_R(X)] = U_R(X)$
(xi) $L_R[L_R(X)] = U_R[L_R(X)] = L_R(X)$

Definition 2.3 ([9]). Let U be the universe, R be an equivalence relation on U and $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$, where $X \subseteq U$ and by the property 2.2, $\tau_R(X)$ satisfies the following axioms:

(i) U and $\phi \in \tau_R(X)$.
(ii) The union of the elements of any sub-collection of $\tau_R(X)$ is in $\tau_R(X)$.
(iii) The intersection of the elements of any finite subcollection of $\tau_R(X)$ is in $\tau_R(X)$.

Therefore, $\tau_R(X)$ is a topology on U called the nano topology on U with respect to X. We call $(U, \tau_R(X))$ as the nano topological space. The elements of $\tau_R(X)$ are called nano open sets (briefly n-open sets). The complement of a nano open set is called a nano closed set (briefly n-closed set).

Definition 2.4 ([2]). Let $(U, \tau_R(X))$ be a nano topological space and $A \subseteq U$. Then A is said to be ng-closed set if $ncl(A) \subseteq B$ whenever $A \subseteq B \subseteq \tau_R(X)$. The complement of an ng-closed set is called a ng-open set.

Definition 2.5 ([11]). Let $(U, \mathcal{N}, I)$ be a nano ideal topological space with an ideal I on U, where $\mathcal{N} = \tau_R(X)$ and $(.)^*_n$ be a set operator from $P(U)$ to $P(U)$ ($P(U)$ is the set of all subsets of U). For a subset $A \subset U$, $A^*_n(I, \mathcal{N}) = \{x \in U : G_n \cap A \notin I$, for every $G_n \in G_n(x)\}$, where $G_n = \{G_n \mid x \in G_n, G_n \in \mathcal{N}\}$ is called the nano local function (briefly, n-local function) of A with respect to I and $\mathcal{N}$. We will simply write A^*_n for $A^*_n(I, \mathcal{N})$.

Theorem 2.6 ([11]). *Let $(U, \mathcal{N})$ be a nano topological space with ideals I, I' on U and A, B be subsets of U. Then*

(i) $A \subseteq B \Rightarrow A^*_n \subseteq B^*_n$,
(ii) $I \subseteq I' \Rightarrow A^*_n(I') \subseteq A^*_n(I)$,
(iii) $A^*_n = n\text{-}cl(A^*_n) \subseteq n\text{-}cl(A)$ (A^*_n *is a nano closed subset of* $n\text{-}cl(A)$),

(iv) $(A_n^*)_n^* \subseteq A_n^*$,
(v) $A_n^* \cup B_n^* = (A \cup B)_n^*$,
(vi) $A_n^* - B_n^* = (A - B)_n^* - B_n^* \subseteq (A - B)_n^*$,
(vii) $V \in \mathcal{N} \Rightarrow V \cap A_n^* = V \cap (V \cap A)_n^* \subseteq (V \cap A)_n^*$ *and*
(viii) $J \in I \Rightarrow (A \cup J)_n^* = A_n^* = (A - J)_n^*$.

Theorem 2.7 ([11]). *If $(U, \mathcal{N}, I)$ is a nano topological space with an ideal I and $A \subseteq A_n^*$, then $A_n^* = n\text{-}cl(A_n^*) = n\text{-}cl(A)$.*

Definition 2.8 ([11]). Let $(U, \mathcal{N})$ be a nano topological space with an ideal I on U. The set operator $n\text{-}cl^*$ is called a nano$*$-closure and is defined as $n\text{-}cl^*(A) = A \cup A_n^*$ for $A \subseteq X$.

Theorem 2.9 ([11]). *The set operator $n\text{-}cl^*$ satisfies the following conditions:*
(i) $A \subseteq n\text{-}cl^*(A)$,
(ii) $n\text{-}cl^*(\phi) = \phi$ *and* $n\text{-}cl^*(U) = U$,
(iii) *If* $A \subset B$, *then* $n\text{-}cl^*(A) \subseteq n\text{-}cl^*(B)$,
(iv) $n\text{-}cl^*(A) \cup n\text{-}cl^*(B) = n\text{-}cl^*(A \cup B)$.
(v) $n\text{-}cl^*(n\text{-}cl^*(A)) = n\text{-}cl^*(A)$.

Definition 2.10 ([11]). An ideal I in a space $(U, \mathcal{N}, I)$ is called $\mathcal{N}$-codense ideal if $\mathcal{N} \cap I = \{\phi\}$.

Definition 2.11 ([11]). A subset A of a nano ideal topological space $(U, \mathcal{N}, I)$ is $n*$-dense in itself (resp. $n*$-perfect and $n*$-closed) if $A \subseteq A_n^*$ (resp. $A = A_n^*$, $A_n^* \subseteq A$).

Lemma 2.12 ([11]). *Let $(U, \mathcal{N}, I)$ be a nano ideal topological space and $A \subseteq U$. If A is $n*$-dense in itself, then $A_n^* = n\text{-}cl(A_n^*) = n\text{-}cl(A) = n\text{-}cl^*(A)$.*

3. On nano-Ig-closed sets and nano-Ig-open sets

Definition 3.1. A subset A of a nano ideal topological space $(U, \mathcal{N}, I)$ is said to be nano-I-generalized closed (briefly, nIg-closed) if $A_n^* \subseteq V$ whenever $A \subseteq V$ and V is n-open.A subset A of a nano ideal topological space $(U, \mathcal{N}, I)$ is said to be nano-I-generalized open (briefly, nIg-open) if $X - A$ is nIg-closed.

Theorem 3.2. *Let $(U, \mathcal{N}, I)$ be a nano ideal topological space. Every ng-closed set is nIg-closed.*

PROOF. Let V be any n-open set containing A. Since A is ng-closed, then $n\text{-}cl(A) \subseteq V$. By Theorem 2.6(iii), we have $A_n^* \subseteq V$. ∎

EXAMPLE 3.3. Let $U = \{a, b, c, d\}$ be the universe, $X = \{a, d\} \subset U$, $U/R = \{\{b\}, \{d\}, \{a, c\}\}$ and $\mathcal{N} = \{U, \phi, \{d\}, \{a, c, d\}, \{a, c\}\}$ and the ideal $I = \phi, \{d\}$. The set $A = \{d\}$ is nIg-closed but not ng-closed.

THEOREM 3.4. *If $(U, \mathcal{N}, I)$ is a nano ideal topological space and $A \subseteq X$, then A is nIg-closed if and only if $n\text{-}cl_n^*(A) \subseteq V$ whenever $A \subseteq V$ and V is n-open in U.*

PROOF. Necessity: Since A is nIg-closed, we have $A_n^* \subseteq V$ whenever $A \subseteq V$ and V is n-open in U. $n\text{-}cl_n^*(A) = A \cup A_n^* \subseteq V$ whenever $A \subseteq V$ and V is n-open in U.

Sufficiency: Let $A \subseteq V$ and V be n-open in U. By hypothesis $n\text{-}cl_n^*(A) \subseteq V$. Since $n\text{-}cl_n^*(A) = A \cup A_n^*$, we have $A_n^* \subseteq V$. ∎

THEOREM 3.5. *If $(U, \mathcal{N}, I)$ is a nano ideal topological space and $A \subseteq X$, then the following are equivalent:*

(i) *A is nIg-closed.*
(ii) *$n\text{-}cl^*(A) \subseteq V$ whenever $A \subseteq V$ and V is n-open in U.*
(iii) *For all $x \in n\text{-}cl^*(A)$, $n\text{-}cl(\{x\}) \cap A \neq \phi$.*
(iv) *$n\text{-}cl^*(A) - A$ contains no nonempty n-closed set.*
(v) *$A_n^* - A$ contains no nonempty n-closed set.*

PROOF. $(i) \Rightarrow (ii)$ If A is nIg-closed, then $A_n^* \subseteq V$ whenever $A \subseteq V$ and V is open in U and so $cl_n^*(A) = A \cup A_n^* \subseteq V$ whenever $A \subseteq V$ and V is open in U. This proves (ii).

(ii) $\Rightarrow$ (iii) Suppose $x \in n\text{-}cl_n^*(A)$ and $n\text{-}cl(\{x\}) \cap A = \phi$. in this case, $A \subseteq X - n\text{-}cl(\{x\})$. We have $n\text{-}cl^*((\{x\}) \subseteq U - (n\text{-}cl(\{x\}))$ and hence $n\text{-}cl^*(A) \cap \{x\} = \phi$. This is a contradiction, since $x \in n\text{-}cl^*(A)$.

(iii) $\Rightarrow$ (iv) Suppose $F \subseteq n\text{-}cl^*(A) - A$, F is n-closed and $x \in F$. Since $F \subseteq U - A, F \cap A = \phi$. We have $n\text{-}cl(\{x\}) \cap A = \phi$ because F is n-closed and $x \in F$. From (iii), this is a contradiction.

(iv) $\Rightarrow$ (v) This is obvious from the definition of $n\text{-}cl^*(A)$.

(v) $\Rightarrow$ (i) Let V be an n-open subset containing A. Since A_n^* is n-closed by means of Theorem 2.6(iii) we obtain $A_n^* \cap (U - V) \subseteq A_n^* - A$ is an n-closed set contained in $A_n^* - A$. By assumption, $A_n^* \cap (U - V) = \phi$. Hence, we have $A_n^* \subseteq V$. ∎

COROLLARY 3.6. *Let $(U, \mathcal{N}, I)$ be a nano ideal topological space and $A \subseteq U$ is an nIg-closed set, then the following are equivalent:*

(i) *A is an $n*$-closed set.*
(ii) *n-$cl^*(A) - A$ is an n-closed set.*
(iii) *$A_n^* - A$ is an n-closed set.*

PROOF. (i) $\Rightarrow$ (ii) If A is $n*$-closed, then n-$cl^*(A) - A = \phi$ and n-$cl^*(A) - A$ is n-closed.

$(ii) \Rightarrow (iii)$ Since n-$cl^*(A) - A = A_n^* - A$, it is clear.

$(iii) \Rightarrow (i)$ If $A_n^* - A$ is n-closed and A is nIg-closed, from Theorem 3.5(v), $A_n^* - A = \phi$ and so A is $n*$-closed. ∎

THEOREM 3.7. *If $(U, \mathcal{N}, I)$ is a nano ideal topological space and A is an $n*$-dense in itself, nIg-closed subset of U, then A is ng-closed.*

PROOF. Suppose A is an $n*$-dense in itself, nIg-closed subset of U. If V is any open set containing A, then $A_n^* \subseteq V$. Since A is $n*$-dense in itself, by Lemma 2.12, n-$cl(A) \subseteq V$ and so A is ng-closed. ∎

COROLLARY 3.8. *If $(U, \mathcal{N}, I)$ is any nano ideal topological space where $I = \{\phi\}$, then A is nIg-closed if and only if A is ng-closed.*

PROOF. The proof follows from the fact that for $I = \{\phi\}$, $A_n^* = n$-$cl(A) \supset A$ and so every subset of U is $n*$-dense in itself. ∎

THEOREM 3.9. *Let $(U, \mathcal{N}, I)$ be a nano ideal topological space. Then every subset of U is nIg-closed if and only if every n-open set is $n*$-closed.*

PROOF. Suppose every subset of U is nIg-closed. If V is n-open, then V is nIg-closed and so $V_n^* \subseteq V$. Hence V is $n*$-closed. Conversely, suppose that every n-open set is $n*$-closed. If $A \subseteq U$ and V is an n-open set such that $A \subseteq V$, then $A_n^* \subset V_n^*$ and so A is nIg-closed. ∎

THEOREM 3.10. *Let $(U, \mathcal{N}, I)$ be a nano ideal topological space. Every $n*$-closed set is nIg-closed.*

PROOF. Let A be a subset of X and A be $n*$-closed. Assume that $A \subseteq V$ and V is n-open. Since A is $n*$-closed, we have $A_n^* \subseteq A$ and so A is nIg-closed. ∎

EXAMPLE 3.11. Let $U = \{a, b, c, d\}$ be the universe, $X = \{a, d\} \subset U$, $U/R = \{\{b\}, \{d\}, \{a, c\}\}$ and $\mathcal{N} = \{U, \phi, \{d\}, \{a, c, d\}, \{a, c\}\}$ and the ideal $I = \phi, \{d\}$. The set $A = \{a, b\}$ is nIg-closed but not $n*$-closed. Since, $A_n^* = \{a, b, c\} \nsubseteq \{a, b\} = A$.

For the relationship related to several sets defined in the paper, we have the following diagram:

$$\begin{array}{ccccc} & & n\text{-closed} & \Longrightarrow & ng\text{-closed} & \Longrightarrow & nIg\text{-closed} \\ & & & & & & \Uparrow \\ n{*}\text{-dense in itself} & \Longleftarrow & & & n{*}\text{-perfect} & \Longrightarrow & n{*}\text{-closed} \end{array}$$

Theorem 3.12. *Let $(U, \mathcal{N}, I)$ be a nano ideal topological space and $A \subseteq U$. If A is $n*$-dense in itself and nIg-closed, then A is ng-closed.*

Proof. Assume A is $n*$-dense in itself and nIg-closed on U. If V is an n-open set containing A, then we have $A^*_m \subseteq V$. Since A is $n*$-dense in itself, Lemma 2.12 implies $n\text{-}cl(A) \subseteq V$ and so A is ng-closed. ∎

Theorem 3.13. *Let $(U, \mathcal{N}, I)$ be a nano ideal topological space and $A \subseteq U$. If A is nIg-closed and n-open then A is $n*$-closed.*

Proof. Let A be n-open. Since A is nIg-closed, we have $A^*_n \subseteq A$. Hence A is $n*$-closed. ∎

Theorem 3.14. *If A and B are nIg- closed , then $A \cup B$ is nIg-closed.*

Proof. Let A and B are nIg-closed sets . Then $A^*_n \subseteq V$ where $A \subseteq V$ and V is n-open and $B^*_n \subseteq V$ where $B \subseteq V$ and V is n-open . Since A and B are subsets of V, $(A^*_n \cup B^*_n) = (A \cup B)^*_n$ is a subset of V and V is n-open. which implies that $(A \cup B)$ is nIg-closed. ∎

Remark 3.15. The Intersection of two nIg-closed sets need not be nIg-closed set which is shown in the following example.

Example 3.16. Let $U = \{a, b, c, d\}$ be the universe, $X = \{b, d\} \subset U$, $U/R =$ $= \{\{c\}, \{d\}, \{a, b\}\}$ and $\mathcal{N} = \{U, \phi, \{c\}, \{d\}, \{a, b, d\}\}$ and the ideal $I = \phi, \{a\}$. Let $A = \{b, d\}$ and $B = \{b, d\}$ be nIg-closed sets. $A \cap B = \{d\}$ is not a nIg-closed set.

Theorem 3.17. *If A is nIg-closed and $A \subseteq B \subseteq A^*_n$, then B is nIg-closed.*

Proof. Let $B \subseteq V$ where V is n-open in $\mathcal{N}$. Then $A \subseteq B$ implies $A \subseteq V$. Since A is nIg-closed, $A^*_n \subseteq V$. Also $B \subseteq A^*_n$ implies $B^*_n \subseteq A^*_n$. Thus $B^*_n \subseteq V$ and so B is nIg-closed. ∎

Theorem 3.18. *Let $(U, \mathcal{N}, I)$ be a nano ideal topological space and $A \subseteq U$. Then A is nIg-open if and only if $F \subseteq n\text{-}int^*(A)$ whenever F is closed and $F \subseteq A$.*

PROOF. Suppose A is nIg-open. If F is closed and $F \subseteq A$, then $U - A \subseteq U - F$ and so $n\text{-}cl^*(U - A) \subseteq U - F$. Therefore, $F \subseteq n\text{-}int^*(A)$. Conversely, suppose the condition holds. Let V be an open set such that $U - A \subseteq V$. Then $U - V \subseteq A$ and so $U - V \subseteq n\text{-}int^*(A)$ which implies that $n\text{-}cl^*(U - A) \subseteq V$. Therefore, $U - A$ is nIg-closed and so A is nIg-open. ∎

THEOREM 3.19. *Let $(U, \mathcal{N}, I)$ be a nano ideal topological space and $A \subseteq U$. Then the following are equivalent.*

(i) *A is nIg-closed.*
(ii) *$A \cup (U - A^*_n)$ is nIg-closed.*
(iii) *$A^*_n - A$ is nIg-open.*

PROOF. $(i) \Rightarrow (ii)$. Suppose A is nIg-closed. If V is any open set such that $(A \cup (U - A^*_n)) \subseteq V$, then $U - V \subseteq U - (A \cup (U - A^*_n)) = A^*_n - A$. Since A is nIg-closed, by Theorem 3.5(v), it follows that $U - V = \phi$ and so $U = V$. Since U is the only open set containing $A \cup (U - A^*_n)$, clearly, $A \cup (U - A^*_n)$ is nIg-closed.

$(ii) \Rightarrow (i)$. Suppose $A \cup (U - A^*_n)$ is nIg-closed. If F is any closed set such that $F \subseteq A^*_n - A$, then $A \cup (U - A^*_n) \subseteq U - F$ and $U - F$ is open. Therefore, $(A \cup (U - A^*_n))^*_n \subseteq U - F$ which implies that $A^*_n \cup (U - A^*_n)^*_n \subseteq U - F$ and so $F \subseteq U - A^*_n$. Since $F \subseteq A^*_n$, it follows that $F = \phi$. Hence A is nIg-closed.

The equivalence of (ii) and (iii) follows from the fact that $U - (A^*_n - A) = A \cup (U - A^*_n)$. ∎

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